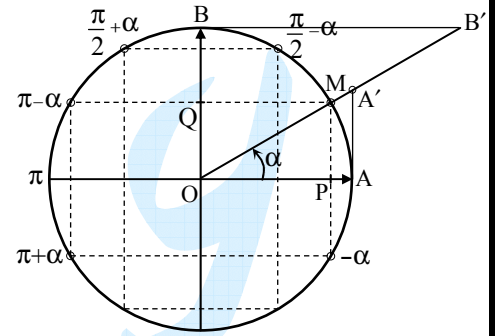


Fonctions Trigonométriques et hyperboliques

α	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin \alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos \alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\operatorname{tg} \alpha$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞

$$\begin{aligned} \cos \alpha &= \overline{OP} & -1 \leq \cos \alpha \leq 1 \\ \sin \alpha &= \overline{OQ} & -1 \leq \sin \alpha \leq 1 \\ \operatorname{tg} \alpha &= \overline{AA'} = \frac{\sin \alpha}{\cos \alpha} = \operatorname{cotg} \alpha \\ \operatorname{cotg} \alpha &= \overline{BB'} = \frac{\cos \alpha}{\sin \alpha} = \operatorname{tg} \alpha \\ \operatorname{seca} &= \frac{1}{\cos \alpha} ; \operatorname{coseca} = \frac{1}{\sin \alpha} \end{aligned}$$



$\sin^2 \alpha + \cos^2 \alpha = 1$

$$1 + \operatorname{tg}^2 \alpha = \frac{1}{\cos^2 \alpha}$$

$$1 + \operatorname{cotg}^2 \alpha = \frac{1}{\sin^2 \alpha}$$

$$\begin{aligned} \cos(\alpha + 2k\pi) &= \cos \alpha \\ \sin(\alpha + 2k\pi) &= \sin \alpha \\ \operatorname{tg}(\alpha + k\pi) &= \operatorname{tg} \alpha \\ \cos(-\alpha) &= \cos \alpha \\ \sin(-\alpha) &= -\sin \alpha \\ \operatorname{tg}(-\alpha) &= -\operatorname{tg} \alpha \end{aligned}$$

• k nombre relatif •

$$\begin{aligned} \cos(\pi + \alpha) &= -\cos \alpha \\ \sin(\pi + \alpha) &= -\sin \alpha \\ \operatorname{tg}(\pi + \alpha) &= \operatorname{tg} \alpha \\ \cos(\pi - \alpha) &= -\cos \alpha \\ \sin(\pi - \alpha) &= \sin \alpha \\ \operatorname{tg}(\pi - \alpha) &= -\operatorname{tg} \alpha \end{aligned}$$

$$\begin{aligned} \cos\left(\frac{\pi}{2} - \alpha\right) &= \sin \alpha \\ \sin\left(\frac{\pi}{2} - \alpha\right) &= \cos \alpha \\ \operatorname{tg}\left(\frac{\pi}{2} - \alpha\right) &= \frac{1}{\operatorname{tg} \alpha} \\ \cos\left(\frac{\pi}{2} + \alpha\right) &= -\sin \alpha \\ \sin\left(\frac{\pi}{2} + \alpha\right) &= \cos \alpha \\ \operatorname{tg}\left(\frac{\pi}{2} + \alpha\right) &= -\frac{1}{\operatorname{tg} \alpha} \end{aligned}$$

$$\begin{aligned} \sin \alpha = 0 &\Leftrightarrow \alpha = k\pi \\ \cos \alpha = 0 &\Leftrightarrow \alpha = \frac{\pi}{2} + k\pi \\ \sin \alpha = 1 &\Leftrightarrow \alpha = \frac{\pi}{2} + 2k\pi \\ \cos \alpha = 1 &\Leftrightarrow \alpha = 2k\pi \\ \sin \alpha = -1 &\Leftrightarrow \alpha = \frac{3\pi}{2} + 2k\pi \\ \cos \alpha = -1 &\Leftrightarrow \alpha = \pi + 2k\pi \end{aligned}$$

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$$\begin{aligned} \cos 2\alpha &= \cos^2 \alpha - \sin^2 \alpha = 2\cos^2 \alpha - 1 \\ &= 1 - 2\sin^2 \alpha = \frac{1 - \operatorname{tg}^2 \alpha}{1 + \operatorname{tg}^2 \alpha} \\ \sin 2\alpha &= 2\sin \alpha \cos \alpha = \frac{2 \operatorname{tg} \alpha}{1 + \operatorname{tg}^2 \alpha} \\ 2\cos^2 \alpha &= 1 + \cos 2\alpha \\ 2\sin^2 \alpha &= 1 - \cos 2\alpha \\ 4\cos^3 \alpha &= \cos 3\alpha + 3\cos \alpha \\ 4\sin^3 \alpha &= -\sin 3\alpha + 3\sin \alpha \end{aligned}$$

$$\begin{aligned} \cos(\alpha + \beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ \cos(\alpha - \beta) &= \cos \alpha \cos \beta + \sin \alpha \sin \beta \\ \sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ \sin(\alpha - \beta) &= \sin \alpha \cos \beta - \cos \alpha \sin \beta \\ \operatorname{tg}(\alpha + \beta) &= \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{1 - \operatorname{tg} \alpha \operatorname{tg} \beta} \\ \operatorname{tg}(\alpha - \beta) &= \frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{1 + \operatorname{tg} \alpha \operatorname{tg} \beta} \\ \operatorname{tg} 2\alpha &= \frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha} \end{aligned}$$

$$\begin{aligned} \cos \alpha = \cos \beta &\Leftrightarrow \begin{cases} \alpha = \beta + 2k\pi \\ \alpha = -\beta + 2k\pi \end{cases} \\ \sin \alpha = \sin \beta &\Leftrightarrow \begin{cases} \alpha = \beta + 2k\pi \\ \alpha = \pi - \beta + 2k\pi \end{cases} \\ \operatorname{tg} \alpha = \operatorname{tg} \beta &\Leftrightarrow \alpha = \beta + k\pi \end{aligned}$$

• k nombre relatif •

$$\begin{aligned} \cos \alpha &= \frac{e^{i\alpha} + e^{-i\alpha}}{2} \\ \sin \alpha &= \frac{e^{i\alpha} - e^{-i\alpha}}{2i} \\ e^{i\alpha} &= \cos \alpha + i \sin \alpha \\ e^{-i\alpha} &= \cos \alpha - i \sin \alpha \\ e &= 2,718281828... \\ i \text{ nbre imag: } &i^2 = -1 \end{aligned}$$

$$\begin{aligned} \sin \alpha \sin \beta &= \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)] \\ \cos \alpha \cos \beta &= \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)] \\ \sin \alpha \cos \beta &= \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)] \\ \operatorname{tg} \alpha \operatorname{tg} \beta &= \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{\operatorname{cotg} \alpha + \operatorname{cotg} \beta} = \frac{\operatorname{tg} \beta - \operatorname{tg} \alpha}{\operatorname{cotg} \alpha - \operatorname{cotg} \beta} \end{aligned}$$

$$\begin{aligned} a \cos \alpha + b \sin \alpha &= \sqrt{a^2 + b^2} \sin(\alpha + \theta) \\ \sin \theta &= \frac{a}{\sqrt{a^2 + b^2}} ; \cos \theta = \frac{b}{\sqrt{a^2 + b^2}} \\ a \cos \alpha + b \sin \alpha &= \sqrt{a^2 + b^2} \cos(\alpha - \theta) \\ \cos \theta &= \frac{a}{\sqrt{a^2 + b^2}} ; \sin \theta = \frac{b}{\sqrt{a^2 + b^2}} \end{aligned}$$

$$\begin{aligned} \sin \alpha + \sin \beta &= 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) \\ \sin \alpha - \sin \beta &= 2 \sin\left(\frac{\alpha - \beta}{2}\right) \cos\left(\frac{\alpha + \beta}{2}\right) \\ \cos \alpha + \cos \beta &= 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) \\ \cos \alpha - \cos \beta &= -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right) \\ \operatorname{tg} \alpha + \operatorname{tg} \beta &= \frac{\sin(\alpha + \beta)}{\cos \alpha \cos \beta} \\ \operatorname{tg} \alpha - \operatorname{tg} \beta &= \frac{\sin(\alpha - \beta)}{\cos \alpha \cos \beta} \end{aligned}$$

$$\begin{aligned} \operatorname{ch} \alpha &= \frac{e^\alpha + e^{-\alpha}}{2} ; \operatorname{sh} \alpha = \frac{e^\alpha - e^{-\alpha}}{2} \\ \operatorname{th} \alpha &= \frac{\operatorname{sh} \alpha}{\operatorname{ch} \alpha} = \frac{e^\alpha - e^{-\alpha}}{e^\alpha + e^{-\alpha}} \\ \operatorname{cth} \alpha &= \frac{\operatorname{ch} \alpha}{\operatorname{sh} \alpha} = \frac{e^\alpha + e^{-\alpha}}{e^\alpha - e^{-\alpha}} \\ \operatorname{ch} \alpha + \operatorname{sh} \alpha &= e^\alpha ; \operatorname{ch} \alpha - \operatorname{sh} \alpha = e^{-\alpha} \\ \operatorname{ch}^2 \alpha - \operatorname{sh}^2 \alpha &= 1 ; \operatorname{th} \alpha \operatorname{cth} \alpha = 1 \\ \operatorname{sh}(-\alpha) &= -\operatorname{sh} \alpha ; \operatorname{ch}(-\alpha) = \operatorname{ch} \alpha \\ \operatorname{th}(-\alpha) &= -\operatorname{th} \alpha ; \operatorname{cth}(-\alpha) = -\operatorname{cth} \alpha \end{aligned}$$

$$\begin{aligned} \operatorname{sh}(\alpha \pm \beta) &= \operatorname{sh} \alpha \operatorname{ch} \beta \pm \operatorname{ch} \alpha \operatorname{sh} \beta \\ \operatorname{ch}(\alpha \pm \beta) &= \operatorname{ch} \alpha \operatorname{ch} \beta \pm \operatorname{sh} \alpha \operatorname{sh} \beta \\ \operatorname{th}(\alpha \pm \beta) &= \frac{\operatorname{th} \alpha \pm \operatorname{th} \beta}{1 \pm \operatorname{th} \alpha \operatorname{th} \beta} \\ \operatorname{sh} 2\alpha &= 2 \operatorname{sh} \alpha \operatorname{ch} \alpha = \frac{2 \operatorname{th} \alpha}{1 - \operatorname{th}^2 \alpha} \\ \operatorname{ch} 2\alpha &= \operatorname{ch}^2 \alpha + \operatorname{sh}^2 \alpha = 2 \operatorname{sh}^2 \alpha + 1 \\ &= 2 \operatorname{ch}^2 \alpha - 1 = \frac{1 + \operatorname{th}^2 \alpha}{1 - \operatorname{th}^2 \alpha} \\ \operatorname{th} 2\alpha &= \frac{2 \operatorname{th} \alpha}{1 + \operatorname{th}^2 \alpha} ; \operatorname{cth} 2\alpha = \frac{1 + \operatorname{cth}^2 \alpha}{2 \operatorname{cth} \alpha} \end{aligned}$$

$$\begin{aligned} \operatorname{sh} \alpha \pm \operatorname{sh} \beta &= 2 \operatorname{sh}\left(\frac{\alpha \pm \beta}{2}\right) \operatorname{ch}\left(\frac{\alpha \mp \beta}{2}\right) \\ \operatorname{ch} \alpha + \operatorname{ch} \beta &= 2 \operatorname{ch}\left(\frac{\alpha + \beta}{2}\right) \operatorname{ch}\left(\frac{\alpha - \beta}{2}\right) \\ \operatorname{ch} \alpha - \operatorname{ch} \beta &= 2 \operatorname{sh}\left(\frac{\alpha + \beta}{2}\right) \operatorname{sh}\left(\frac{\alpha - \beta}{2}\right) \\ \operatorname{th} \alpha \pm \operatorname{th} \beta &= \frac{\operatorname{sh}(\alpha \pm \beta)}{\operatorname{ch} \alpha \operatorname{ch} \beta} \\ (\operatorname{ch} \alpha \pm \operatorname{sh} \alpha)^n &= \operatorname{ch} n\alpha \pm \operatorname{sh} n\alpha \end{aligned}$$